# Optimal Control Applied to the Transmission Investment Strategy Problem

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**Abstract**: In the deregulated competitive electric power marketplace, the transaction price and quantity are decided by the supply and demand. With the growth of power demand in recent years, the limit of transmission capacity becomes an obstacle to competitive transactions. This moves the market solution away from the ideal competitive solution, and brings deadweight losses that must be carried by society. In order to maximize total social welfare and achieve economic efficiency, the market regulator must build some new transmission facilities, e.g., new lines or power flow controller. If the economic cost is less than the increase of trading surplus, (i.e., the benefit), then the investment is worthy. This paper discusses investment strategies for market regulators deciding to invest in new transmission facilities in a growing competitive power market. We can see in such a market, the economic profit and cost from one period to another can be represented by a series of discrete-time dynamic equations, and the objective function is in quadratic form of state and control variables. After constructing models, the decision making can be viewed as a LQR (Linear Quadratic Regulator) problem and the discrete-time Ricatti equation can be applied to solve it.

**Key Word:** FACTS, optimal control, linear quadratic regulator problem, Ricatti equation, transmission expansion planning

## 1 Introduction

Deregulation is a very popular topic in the electric power industry in recent years because it dramatically changes the way the power system is operated. It helps to break the monopoly and create a competitive market. In the market assumed for this research, both sellers and buyers are price takers which means they are unable to control the price. Some effective mechanism, e.g., auction, will help to set prices which will provides incentives for increasing the system efficiency. A distinguishing feature of the new market is the separation of generation, distribution and transmission. In the traditional vertically integrated monopoly, generation, distribution and transmission facilities were owned by utilities. That gave the utilities a competitive advantage over other generators. The utilities are now presented directly a large number of potential consumers each having a demand curve. Associated with their own cost curve and transmission capacity between the supplier and demander, they will choose their generation level and set the price knowing that they are in competition with other generators. In the deregulated market, transmission capacity at any given time is one of the unknown parameters to generation companies and it is no longer an effect on the generation pattern. Both sellers and buyers will face the market price and adjust their generation/consumption level according to this price, e.g., let their marginal cost/revenue equal to this price. Then, through the competitiveness, the near true cost/revenue curve can be found. The goal of the deregulation is lower electricity prices and more efficient use of the electrical infrastructure. This translates into increases in the amount of power; hence transmission capacity becomes a very important issue.

In the deregulated market, the transmission grids are independently owned and regulated by state to provide open access for all power producers and consumers. The thermal constraints of the lines often prevent buyers from buying cheaper electricity and the long lines cause heavy losses in transmission. Both of these factors indirectly contribute to dead weight economic losses and cause inefficiency to the market. In many electric markets, the regulated transmission system does not encourage nor allow the transmission owners to expand the system as needed. In these systems, the capital-intensive decisions to improve the network may be made by central planners or other independent entities. In any case, with the growth of power transactions, the market regulator or designee must invest on the expansion of transmission system.

There are many ways to expand the transmission system. To build new transmission lines or to re-conductoring the existing lines is the most straightforward way. Reactive power compensation devices, e.g., shunt capacitor/reactor, series capacitor, SVC, SVG, can change the properties of the transmission grids and alter the Jacobian matrix in the power flow equations. Thus the operating pattern of the system is changed after the compensation. Recently, many concerns have been paid to FACTS (Flexible AC Transmission System), a power electronic/ microprocessor-based fast switching compensation device which can which can regulate the power flow with great flexibility and increase the dynamic properties of the power system [1,2].

Although there are many papers discussing the technical issues about the expansion of transmission system, not enough concern is given to the economical issues. M.D. Ilic reviewed the role of the transmission grid in achieving high efficiency in deregulated market [3]. She also provided a global assessment of the planning and operating problem for power transmission grids in the future. J. Mutale et al. gave an economic assessment of competitiveness of FACTS devices against network reinforcement [4]. Modern control theory is widely used in the analyzing of power system market problem. F. L. Alvarado discussed the economic stability of simple power system markets using linear control theory [5].

The focus of this paper is to find the optimal investment strategies for market regulators on transmission expansion in a competitive, growing demand market. Section 2 gives the mathematical dynamic model of the market. The model assumes a fixed supply curve and a growing demand curve. In the subsections, the objective function, the discrete-time dynamic model of demand, investment and economic cost are discussed separately. Section 3 discusses the application of discrete-time Ricatti equation on solving the optimal problem. Finally section 4 presents some conclusion and lists areas of future work that the authors intend to investigate.

## 2 Mathematical Investment Model for Transmission Expansion

In the competitive marketplace, the transaction price and quantity is decided by supply and demand. From the regulator's point of view, social welfare is to be maximized. Because of the bottleneck of transmission capacity, Pareto optimality may not be achieved and dead weight losses will result for society. Installing new transmission lines or power flow regulator, e.g., FACTS devices can increase the market volume, and hence increase the total profit.

We assume a highly competitive deregulated marketplace in which both sellers and buyers are price takers and attempt to maximize their own profit. We assume the price taker will rationally continue to produce/consume electricity (assuming they are able to find someone with which to contract) until their marginal cost/revenue is equal to the equilibrium price. If they do this, their profit is to be maximized. The cost curve for sellers, e.g., GENCO (generation company), is a function of power generated and is commonly given in the following quadratic form [6]:

$$C(P) = 0.5 a_s P^2 + b_s P + c_s$$

Here, $a_s$, $b_s$, $c_s$, are constants and P is the power generated. Then the marginal cost for GENCO is:

$$MC(P) = a_s P + b_s$$

For buyers, e.g., ESCO (Energy Service Company), although there is not an explicit form of the revenue curve, they can build a pseudo curve to model the revenue versus the amount of power they sold to the consumer. The revenue curve should be quadratic and concave with downward sloping linear rate of revenue. The decremental revenue curve is:

$$R(P) = -0.5 a_d P^2 + b_d P + c_d$$

Same as the cost curve, $a_d$, $b_d$, $c_d$ here are constants and P is the power generated. Then the marginal revenue of the ESCO is also a negative-slope linear function of the amount:

$$MR(P) = -a_d P + b_d$$

In the auction market, GENCO/ESCO will bid based on their marginal cost/revenue plus some profit. [9] According to the economic theory, the long-term economic profit of the market participants will be zero under perfect competition. If the number of players is large enough, the aggregate marginal cost (revenue curve) will converge to the "true" supply (demand curve) of the market.

Assume the "true" supply and demand curves are found by auction and they are exact linear. The market price and transaction amount should be the intersection point of the supply and demand curve. But because of transmission capacity limitations, the transaction quantity can not be achieved and the price will be higher and transaction amount will be less than under the perfectly competitive scenario. This will lead to deadweight losses.

As shown in figure 1, S(P) and D(P) are supply and demand curve respectively. They are both linear function of the transaction quantity P. $P^{com}$ is the ideal transaction quantity in competitive market and the actual transaction quantity is $P^{max}$. The total trading surplus with the limited transmission is:

$$\pi = \int_0^{P_{max}} [D(P) - S(P)] dP$$

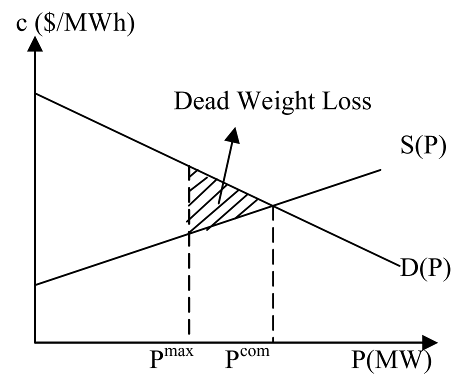


Fig.1 Competitive market

In order to realize the potential profit of the market, the regulator wants to build new lines to increase line capacity or add power flow controllers to increase the network capability. Prior to deciding on a course of action, the wise system planner should do some research on the investment options to determine whether they are feasible and profitable and to decide the best invest strategy.

To solve such a problem it is necessary to make some assumptions. The following are the major assumptions we make in this work:

- The supply and demand curve is approximated by a linear function of the transaction quantity.
- The demand for power is fixed in one planning period (e.g., year) then may change in the next planning period.
- The demand elasticity of price is constant.
- Sufficient supply capacity exists and the supply curve is unchanging.
- Losses in the transmission may be ignored.
- The interest rate of saving or other investment is unchanging during one planning years and may change in the next year.

Here is a list of parameters and their meanings that have been used in this paper:

| | |
|---|---|
| $P_k^{max}$ | The maximum transaction capacity of the market in time k. |
| $I_k$ | The investment of new lines or power flow controller in time k. |
| $M_k$ | The economic cost in time k, equal to the sum of explicit cost (investment on new lines and/or power flow controller) and the implicit cost (interest from bank or other investment). |
| $\tau_k$ | The interest rate of bank or other investment in time k. |
| $P_k$ | Transaction quantity in time k. |
| $\varepsilon_k$ | The growth rate of the demand in the k$^{th}$ planning years. |
| $T_k$ | The trading surplus in time k. |
| $\xi_k$ | The first order derivative of $P^{max}$ with respect to investment $I$ in time k. |
| $U_k$ | The cost for using control in time k. |
| $a_s$, $b_s$ | The slope rate and constant term of the supply curve. |
| $a_d$ | The slope rate of the demand curve. |
| $b_{dk}$ | The constant term of the demand curve in time k, (this corresponds to the linear term of the quadratic cost curve). |

*2.1 Demand and Supply Curve*

As mentioned earlier, this work assumes that both the demand and the supply curve are linear function of transaction amount. This work further assumes that the range of demand and supply are from 0 to a fixed maximum number. The supply curve in time k is given by: $S_k = a_s P_k + b_s$. We assume that it will be constant over the planning years and there is sufficient supply to satisfy the demand. The demand curve in time k is: $D_k = a_d P_k + b_{dk}$. Demand is assumed to change in the planning years and the price elasticity $a_d$ is assumed to be unchanged. So only the constant part of the demand curve changes. Then we have a dynamic system equation as:

$$b_{d,k+1} = \varepsilon_k b_{dk} \qquad (1)$$

Note here that the change of power demand can be positive, negative or zero. This is reflected by the value of $\varepsilon_k$.

In fact, the demand oscillates during different times of the day and different seasons of the year. It's very hard to build an exact function to express the demand. The paper simplifies the problem and excludes the exogenous variables from the equation. A commonly included non-stochastic random disturbance term is also ignored.

*2.2 The Trading Surplus in the Transaction*

If generator A was willing to sell electricity at a minimum of \$10/MWh (its valuation is \$10/MWh) and the price is \$15/MWh, the generator's surplus is \$5/MWh times the quantity of power they sold. Producer surplus is commonly called profit, whereas the surplus experienced by the buyer is commonly termed consumer surplus. Together, the profit and the consumer surplus make up the trading surplus. In time k, the trading surplus $T^k$ can be calculated easily as:

$$\begin{aligned} T_k &= \int_0^{P_k^{max}} [D(P) - S(P)]dP \\ &= \int_0^{P_k^{max}} [(a_d P_k + b_{dk}) - (a_s P_k + b_s)]dP \\ &= \frac{1}{2}(a_d - a_s)P_k^{max\,2} + (b_d^k - b_s)P_k^{max} \qquad (2) \end{aligned}$$

Thus the trading surplus is represented by a quadratic function of the maximum transaction amount.

*2.3 Investment Effect on Transaction Limits*

If the regulator or system planner at the ISO/ICA approves investment of building new lines, increasing line limits through re-conductoring or the addition of a power flow controller, the volume of market should be increased. Assume this increase is linear to the money invested, then we get:

$$P_{k+1}^{max} = P_k^{max} + \xi_k I_k \qquad (3)$$

The capacity of market can be calculated in a manner similar to the calculation of ATC (availabe transmission capability) between two nodes. In fact, the maximum transaction capability is also affected by the transaction pattern and the base case running point. It is difficult to determine how much will it cost to increase $P^{max}$ by one unit. For simplicity, this work just assumes $P^{max}$ is also a linear function of the investment *I*.

### 2.4 Economic Cost

The money spent on re-conductoring building new lines or adding power flow controller is the direct cost or explicit cost. If the system operator saves money in a bank or makes some other investment, they will receive other funds, e.g., saving account interests. This part of money is actually paid indirectly and it is the so-called implicit cost. The total economic cost includes both explicit and implicit cost. So the economic cost in time interval (k+1) is:

$$M_{k+1} = (1+\tau)M_k + I_k \qquad (4)$$

The economic cost is assumed to be calcualated in the begining of each year and before the investment. Then it doesn't include the investment of this year. So the economic cost in period k is equal to economic cost of previous year plus some interest plus the investment of previous year. This will make the equations neat. Only when the profit of building new lines or adding power flow controller is larger than the return on bank savings will they make the investment.

### 2.5 System Equations

Combining equations (1), (2) and (4), we can get the system equations as:

$$\begin{bmatrix} P_{k+1}^{max} \\ b_{d,k+1} \\ M_k \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \varepsilon_k & 0 \\ 0 & 0 & 1+\tau \end{bmatrix} \cdot \begin{bmatrix} P_k^{max} \\ b_{d,k} \\ M_k \end{bmatrix} + \begin{bmatrix} \xi_k \\ 0 \\ 1 \end{bmatrix} I_k \qquad \text{-(5)}$$

It's a set of dynamic discrete-time equations. The values of state variables in a time depend on the value of state variables and the control variables in previous time.

### 2.6 The Objective Function

The objective function is the economic profit minus the control cost. The economic profit is defined as the trading surplus minus the economic cost. Let $\pi_k$ denotes the economic profit, then:

$$\begin{aligned} \pi_k &= T_k - M_k \\ &= \frac{1}{2}(a_d - a_s){P_k^{max}}^2 + (b_{d,k} - b_s)P_k^{max} - M_k \end{aligned}$$

The economic cost of the investment $M_k$ is one part of the control cost and it has already been considered in the economic profit. Besides that, there are also other implicit cost. That contains the operating/maintenance cost of the new transmission facilities and the difficulty in gathering money. In this research, this part of cost is modeled by the quadratic form of the investment:

$$U_k = rI_k^2$$

Then the objective function is:

$$J = \sum_{k=1}^{N} (\pi_k - U_k)$$

### 2.7 Converting the Problem to LQR Form

The LQR (Linear-Quadratic Regulator) problem is defined as to find the optimal control sequence for the linear systems with quadratic performance indices. Now the system equations are linear and we want to find a performance index (objective function) in quadratic form of the state and control variables. Introduce an artificial variable $y$ and define:

$$y_{k+1} = y_k, \quad y_0 = 1.$$

We can see that $y$ will be 1 over the planning time. After introducing the artificial variable, the system dynamic equation (equation 5) becomes:

$$\begin{bmatrix} P_{k+1}^{max} \\ b_{d,k+1} \\ M_{k+1} \\ y_{k+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \varepsilon_k & 0 & 0 \\ 0 & 0 & 1+\tau & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} P_k^{max} \\ b_{d,k} \\ M_k \\ y_k \end{bmatrix} + \begin{bmatrix} \xi_k \\ 0 \\ 1 \\ 0 \end{bmatrix} I_k$$

Now define:

$$x_k = \begin{bmatrix} P_k^{max} \\ b_{d,k} \\ M_k \\ y_k \end{bmatrix}, A_k = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \varepsilon_k & 0 & 0 \\ 0 & 0 & 1+\tau & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, B_k = \begin{bmatrix} \xi_k \\ 0 \\ 1 \\ 0 \end{bmatrix}, u_k = I_k$$

here, $x_k$ is the state variable and $u_k$ is the control variable. The system equation can be written in short as:

$$x_{k+1} = A_k x_k + B_k u_k$$

which is a set of linear, discrete-time dynamic equations.

With the artificial variable, the economic profit can be written in quadratic form as:

$$\pi_k = \begin{bmatrix} P_k^{max} \\ b_{d,k} \\ M_k \\ y_k \end{bmatrix}^T \begin{bmatrix} .5(a_d - a_s) & .5 & 0 & -.5b_s \\ .5 & 0 & 0 & 0 \\ 0 & 0 & 0 & -.5 \\ -.5b_s & 0 & -.5 & 0 \end{bmatrix} \cdot \begin{bmatrix} P_k^{max} \\ b_{d,k} \\ M_k \\ y_k \end{bmatrix} \qquad \text{-(6)}$$

We can define:

$$Q = \begin{bmatrix} .5(a_d - a_s) & .5 & 0 & -.5b_s \\ .5 & 0 & 0 & 0 \\ 0 & 0 & 0 & -.5 \\ -.5b_s & 0 & -.5 & 0 \end{bmatrix} \qquad \text{-(7)}$$

Then the objective function is:

$$Max \quad \sum_{i=1}^{N} \pi_k = \sum_{i=1}^{N} (x_k^T Q x_k - rI_k^{\,2}) \qquad \text{-(8)}$$

After the manipulation, the problem has a quadratic objective function and a set of linear system equations. It's a free final state, fixed horizon period LQR problem. The discrete-time Ricatti equation can be applied directly to solve the problem.

## 3 Discrete-Time Ricatti Equation

The discrete-time LQ regulator with free final state is formulated as:

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System Equation: $x_{k+1} = A_k x_k + B_k u_k$

Performance Index:

$$J = \frac{1}{2} x_N^T S_N x_N + \frac{1}{2} \sum_{k=1}^{N} (x_k^T Q_k x_k + u_k^T R_k u_k)$$

Assumption: $S_N \geq 0, Q_k \geq 0, R_k > 0,$ $S_N$ given, all three matrices are symmetric

Optimal Feedback Control:

$$S_k = A_k^T (S_{k+1}^{-1} + B_k R_k^{-1} B_k^T)^{-1} A_k + Q_k \quad (k < N)$$

*-Ricatti Equation*

$$K_k = (B_k^T S_{k+1} B_k + R_k)^{-1} B_k^T S_{k+1} A_k$$

*-Kalman Gain*

$$u_k^* = -K_k x_k$$

*-Optimal Control Sequence*

Note: Subscript k in all the above equations denotes time k. The detailed derivation can be found in [7].

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This results in a time-varying state-variable feedback control. The current control vector depends on the current state. The final state weight matrix $S_N$ is known, we can calculate $S_{N-1}$, $S_{N-2}$, ..., $S_1$ backward in time. The Kalman gain matrices $K_k$ do not depend on the state variables. So it can be computed offline and stored in the memory of computer before the control is applied to the system.

For the problem presented in this paper, there is no cost coefficient for the final state, so $S_N=0$. The cost coefficient for control variable is $R=r$. The LQR problem is to minimize the performance index, whereas the problem which interests us is the maximization of the objective function. We can reverse the sign of equation (8) and get:

$$Min \quad \sum_{i=1}^{N} \pi_k = \sum_{i=1}^{N} [x_k^T (-Q) x_k + I^k r I^k]$$

So the cost coefficient for the state variables in the middle states is -$Q$ which is independent of time. It's easy to demonstrate that -$Q$ is semi-positive definite and symmetric. The cost coefficient of investment r is greater than zero. The initial values of the problem are: $P_0^{max}$ and $b_{d,k}$ are known fixed numbers, $M^k=0$, $x^k=1$. So all the requirements of LQR problem are met and the result of optimal control sequence can be used directly.

## 4 Conclusion and Future Work

This paper presents the application of modern optimal control theory to the problem of transmission investment strategies from the point of view of the power market regulator. With the growth of the social economy, the demand for power is changing from day to day and the new power generators will continue to emerge in this profitable and competitive market. In order maintain sufficient transmission capacity capable of supporting the power market growth, the market planner had better consider market volume and plan for transmission expansion. This paper builds a brief model of the dynamic market to help the system planner make an informed investment decision. The system equations are linear and the objective function is in quadratic form of the state variables and control variable. It's actually an LQR problem and the discrete-time Ricatti equation can be applied directly.

There are many assumptions in the problem as it is formulated in this paper. Future work should remove some of the restrictions and build a more general model. The following are areas that the authors have specifically targeted for work in the future:

- Shape of the supply and demand curve. Both the supply and the demand will change in the market and the shape of the curves may change. Exogenous variables like season, time also should be included in the model, so is the random disturbance term. Then the problem will become a stochastic control problem.
- Market volume. The market volume is different under different transaction cases and different initial conditions. To make the model more complete, more variables should be included, e.g., the operating point of the generators before the investment. Because the operating point of the generators changes in every hour, we can build a simple regression model and calculate the expected value of the operating point of each generator in one year. Based on this we can build a

relationship between market volume and new transmission facilities. Of course the disturbance terms should be included.

- Cost coefficient of the control variable (investment). Maybe the quadratic form can not exactly represent the constraint for the investment. If the cost of investment, excluding the economic cost (investment itself and potential interest), is written as: $U_k = I_k^{\alpha}$. Combining the operating cost, maintenance cost and the happiness level of the regulator, $1<\alpha<2$ will be reasonable. The reason why $\alpha>1$ is the unwillingness of the regulator to do investment. We choose $\alpha=2$ for the simplicity of calculation in this paper.
- Because the LQR is based on backward-in-time calculation, it is very important to know all the parameters over the planning years, e.g., the growth rate in 10 years, the supply/demand curve in 10 years, etc. But it is very difficult to forecast the change of market in future years. Many statistics must be used to build the forecast model. The accuracy of the forecast will have large effect on the optimality of the investment strategy.

The complete system model and the objective function may contain information from economics, statistics and topology of the power system. The application of optimal control theory should be very beneficial to the market regulators in identifying transmission strategies that meet the needs of the market participants, and minimize social costs.

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## 6 Biographies

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